References

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Explicit Finite-Difference Method for Calculating Laminar and Turbulent Flows

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The purpose of this note is to describe an explicit finite-difference program developed at Boeing[1] which has been applied successfully to laminar and turbulent wake and free shear layer problems with finite rate or equilibrium chemistry. Two formulations have been used, one in the physical plane and the other in the von Mises plane. Both are written using forward-difference approximations for the derivatives in x and average-difference approximations for derivatives in r or ψ.

It was found that the physical plane formulation was adequate for the calculation of wakes. However, for shear layer problems, the physical plane formulation had to be abandoned because of stability problems. The von Mises plane formulation proved adequate in this case.

Run times (including print-out) on an IBM 7090 for typical shear layer and wake problems range from 5 to 30 min, depending on the type of chemistry and input flow conditions. It was found that instability usually starts with the energy equation, and the stability parameter is an order of magnitude less than that given by Wu[2] which was derived for the momentum equation. It was found that restrictions placed on step size by convergence and stability considerations were not prohibitive for many flow conditions of interest.

Some typical results are shown in Figs. 1 and 2. Figures 1a–1c present the results of a calculation of the development of a two-dimensional free mixing zone between air and a combustible gas with the following composition (in mass fractions): H_2O, 0.2880; H_2, 0.0124; O_2, 0.0106; CO_2, 0.2630; and CO, 0.4250. Two reactions were considered:

$$2H_2 + O_2 \rightleftarrows 2H_2O$$

$$2CO + O_2 \rightleftarrows 2CO_2$$

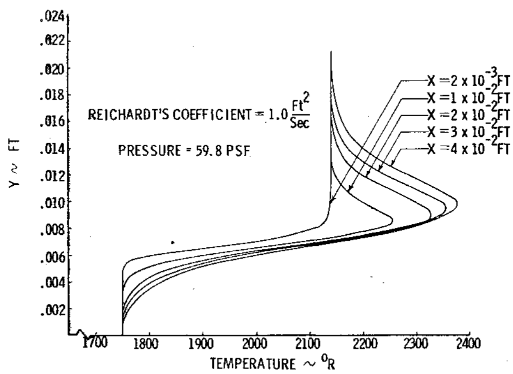

Fig. 1a Thermal profiles for the development of a two-dimensional turbulent mixing zone with combustion.

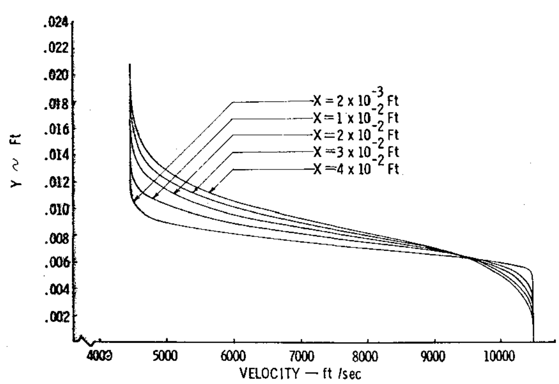

Fig. 1b Velocity profiles for a turbulent, combustible, mixing layer.

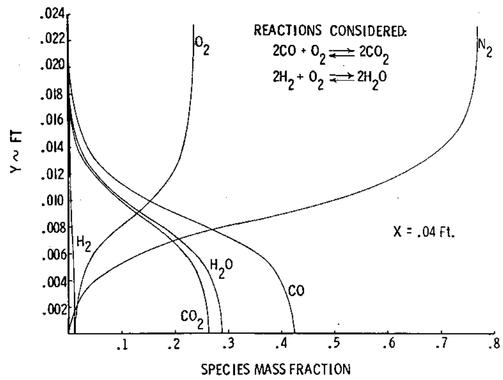

Fig. 1c Species mass fraction profiles for a turbulent mixing layer with combustion.

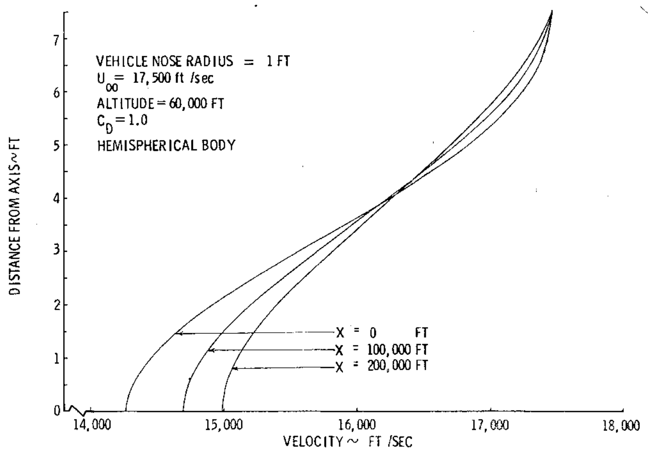

Fig. 2a Velocity profiles for a laminar, equilibrium far wake.

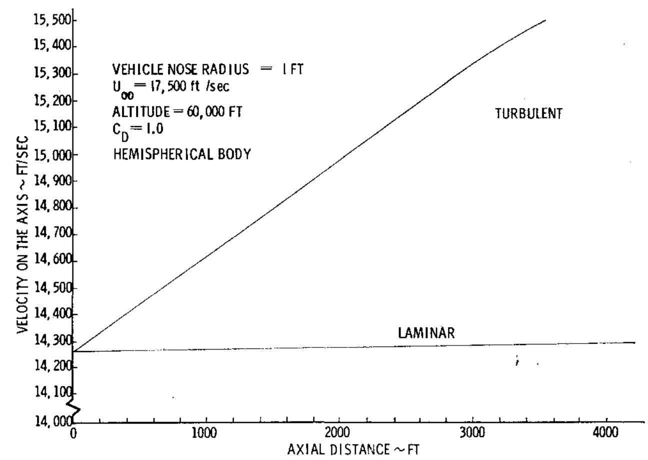

Fig. 2b Comparison of laminar and turbulent axial velocity distributions for an equilibrium wake.

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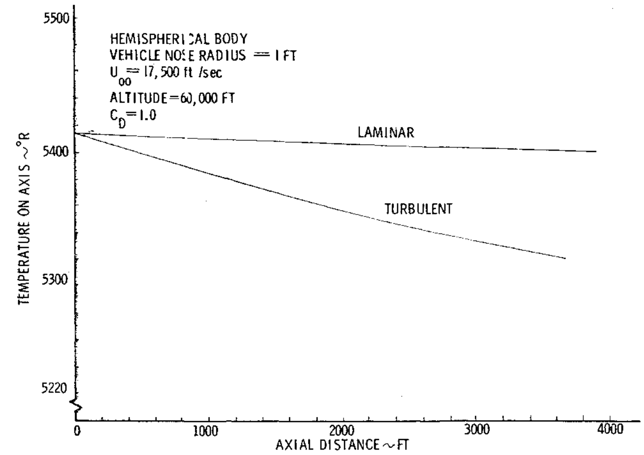

Fig. 2c Comparison of laminar and turbulent axial temperature distributions for an equilibrium wake.

The development of the combustion zone is clearly evident in the temperature profiles.

Figures 2a–2c show typical results for a fully laminar and fully turbulent axisymmetric wake. The program presently is being modified in an attempt to calculate boundary-layer flows with injection and gas-particle flows.

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Influence of Wall Conductance on MHD Energy Conversion

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THE purpose of this note is to examine the influence of finite electrical conductivity walls normal to the magnetic field in a constant area MHD generator configuration. Although the nonelectrode walls of an MHD generator are nominally insulators at room temperature, it is likely that, at the elevated operating temperatures, they will become slightly conducting.

In order to focus attention on the influence of the wall properties, certain simplifying assumptions will be made. The flow is assumed to be in the x direction, laminar, steady, with constant properties. A constant magnetic field is applied in the y direction, and the z dimension of the channel is large compared to the y dimension so that property changes in the z direction are negligible. The nonelectrode walls have thicknesses d_1 and d_2 and electrical conductivities σ_1 and σ_2. Hall effects are neglected so that simple Ohm's law is applicable.[1,2]

The equation of motion and the simplified form of Ohm's law appropriate to the present problem are

$$-\frac{dp}{dx} + \mu\frac{d^2u}{dy^2} - J_zB_y = 0 \tag{1}$$

$$J_z = \sigma(E_z + uB_y) \tag{2}$$

where E_z is constant and has the same value in the fluid and walls. It is convenient to introduce the following dimensionless parameters:

$$\eta = y/w \qquad \alpha = x/w \qquad \gamma = u/\bar{u}$$

$$\pi = \frac{p}{(\mu\bar{u}/w)} \qquad M = B_yw\left(\frac{\sigma}{\mu}\right)^{1/2} \qquad \Phi = \frac{E_z}{B_y\bar{u}} \tag{3}$$

where the reference velocity $\bar{u}$ is the mean velocity defined as

$$\bar{u} = \frac{1}{2w}\int_{-w}^{w} udy \tag{4}$$

The solution to Eq. (1) is the well-known Hartmann profile

$$\gamma = \left(\frac{M}{M - \tanh M}\right)\left(1 - \frac{\cosh M\eta}{\cosh M}\right) \tag{5}$$

The total current per unit length in the x direction, including leakage current in the nonelectrode walls, is given by

$$I = \sigma_1\int_{-(w+d_1)}^{-w} E_zdy + \sigma\int_{-w}^{w}(E_z + uB_y)dy + \sigma_2\int_{w}^{w+d_2} E_zdy \tag{6}$$

or

$$I^* = 2 + \Phi(2 + \theta_1 + \theta_2) \tag{7}$$

where

$$I^* = \frac{I}{\sigma\bar{u}B_yw} \qquad \theta_1 = \frac{\sigma_1d_1}{\sigma w} \qquad \theta_2 = \frac{\sigma_2d_2}{\sigma w} \tag{8}$$

The quantities θ_1 and θ_2 are the conductances of the lower and upper walls, respectively.

The quantity $2 + \theta_1 + \theta_2$ can be given a simple physical interpretation as follows. If L is the width of the channel in the z direction, then $R_i = L/2\sigma w$ represents the internal resistance of a fluid element of unit length in the x direction and of height $2w$. Likewise $R_1 = L/\sigma_1d_1$ and $R_2 = L/\sigma_2d_2$ are the resistances of the channel walls. A simple manipulation shows that

$$2 + \theta_1 + \theta_2 = 2(R_i/R^*) \tag{9}$$

where

$$R^* = \left(\frac{1}{R_i} + \frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \tag{10}$$

Thus R^* is the effective resistance of the three parallel resistances R_i, R_1, R_2, and $2 + \theta_1 + \theta_2$ is proportional to the ratio of fluid resistance to total resistance consisting of the parallel resistances of fluid and channel walls.

If V is the voltage difference across the channel in the z direction, then $V = -LE_z$. In MHD generator studies, it is convenient to work with a parameter expressing the ratio of operating voltage to open circuit voltage. Thus one defines

$$K = \frac{V}{V_{\text{open}}} = \frac{E_z}{E_z)_{\text{open}}} = \frac{\Phi}{\Phi_{\text{open}}} \tag{11}$$

where Φ_{open} is obtained from Eq. (7) by setting $I^* = 0$. The power output per unit length in the x direction is given by the expression $\mathcal{P} = IV$. Substituting for I from Eqs. (7) and

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